



## ORIGINAL RESEARCH PAPER

## Distribution of energy in propagation for ocean extreme wave generation in hydrodynamics laboratory

D. Fadhiliani<sup>1</sup>, M. Ikhwan<sup>1</sup>, M. Ramli<sup>2,\*</sup>, S. Rizal<sup>3</sup>, M. Syafwan<sup>4</sup><sup>1</sup>Graduate School of Mathematics and Applied Sciences, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia<sup>2</sup>Department of Mathematics, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia<sup>3</sup>Department of Marine Sciences, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia<sup>4</sup>Department of Mathematics, Universitas Andalas, Padang 25163, Indonesia

## ARTICLE INFO

## Article History:

Received 18 March 2021

Revised 28 May 2021

Accepted 29 June 2021

## Keywords:

Benjamin Bona Mahony (BBM)

Energy

Extreme wave

Hamiltonian

Ocean wave

Offshore structure

## ABSTRACT

**BACKGROUND AND OBJECTIVES:** The hydrodynamic uncertainty of the ocean is the reason for testing marine structures as an initial consideration. This uncertainty has an impact on the natural structure of the topography as well as marine habitats. In the hydrodynamics laboratory, ships and offshore structures are tested using mathematical models as input to the wave marker. For large wavenumbers, Benjamin Bona Mahony's equation has a stable direction and position in the wave tank. During their propagation, the generated waves exhibit modulation instability and phase singularity phenomena. These two factors refer to Benjamin Bona Mahony as a promising candidate for generating extreme waves in the laboratory. The aim of this research is to investigate the distribution of energy in each modulation frequency change. The Hamiltonian formula that describes the phenomenon of phase singularity is used to observe energy. This data is critical in determining the parameters used to generate extreme waves.

**METHODS:** The envelope of the Benjamin Bona Mahony wave group can be used to study the Benjamin Bona Mahony wave. The Benjamin Bona Mahony wave group is known to evolve according to the Nonlinear Schrodinger equation. The Hamiltonian governs the dynamics of the phase amplitude and proves the Nonlinear Schrodinger equation's singularity for finite time. The Hamiltonian is derived from the appropriate Lagrangian for Nonlinear Schrodinger and then transformed into the Hamiltonian  $H(G, \phi)$  with the displaced phase-amplitude variable.

**FINDINGS:** Potential energy is related to wave amplitude and kinetic energy is related to wave steepness in the study of surface water waves. When  $\tilde{\nu}=0.5$ , the maximum wave amplitude and steepness are obtained. When  $\tilde{\nu}>0.5$ , extreme waves cannot be formed due to steepness. This is due to the possibility of breaking waves into smaller waves on the shore. In terms of position, the energy curve is symmetrical.

**CONCLUSION:** According to Hamiltonian's description of the energy distribution, the smaller the modulation frequency, the greater the potential and kinetic energy involved in wave propagation, and vice versa. While the wave's amplitude and steepness will be greatest for a low modulation frequency, and vice versa. The modulation frequency considered as an extreme wave generator is  $\tilde{\nu}=0.5$ , because the resulting amplitude is quite high and the energy in the envelope is also quite large.

DOI: [10.22034/gjesm.2022.01.02](https://doi.org/10.22034/gjesm.2022.01.02)

©2022 GJESM. All rights reserved.



NUMBER OF REFERENCES

37



NUMBER OF FIGURES

7



NUMBER OF TABLES

0

\*Corresponding Author:

Email: [marwan.math@unsyiah.ac.id](mailto:marwan.math@unsyiah.ac.id)

Phone: +62 813-9766-8376

ORCID: [0000-0003-1225-9063](https://orcid.org/0000-0003-1225-9063)

Note: Discussion period for this manuscript open until April 1, 2022 on GJESM website at the "Show Article."

## INTRODUCTION

Mathematical equations have been used to investigate marine structure tests (Samaras and Karambas, 2021; Ikhwan et al., 2021). This test ensures that the building will not be damaged by high-energy waves and currents. High-energy waves have an impact on the natural structure of the topography as well as marine habitats. Extreme waves are based on events that have occurred and are scientifically recorded tests (Didenkulova, 2020). Activities in the hydrodynamics laboratory necessitate the use of mathematical models for water surface waves. The water waves created in this laboratory are used to assess the durability of ship and offshore structure models. There are several mathematical models that have been used including Korteweg de Vries (KdV) (Horn et al., 1999; Karjanto et al., 2010) and Benjamin Bona Mahony (BBM) (Shiralashetti and Hanaji, 2021). However, the wave that will be generated has high characteristics and will not break, which is known as an extreme wave (Waseda et al., 2012; Zhao et al., 2010). Given the limitations of the laboratory's current testing pool, this characteristic is required. When the height of a wave exceeds 2.2 times that of the average wave, it is considered extreme (Dean, 1990). Following the simplification of the Boussines Equation, KdV and BBM, have only one direction of propagation. Korteweg de Vries was the first to propose the KdV equation (Korteweg et al., 1895). A study of KdV revealed that the resulting waveform was unstable for large  $k$  wavenumbers (Myint-U et al., 2007; Wang et al., 2018). The characteristics of the waves generated in the hydrodynamic laboratory pond, on the other hand, must have a short wavelength or a large wave number. These flaws appear to be overcome by the BBM equation (Benjamin et al., 1972). BBM wave is useful as initial wave because it does not propagate in short waves with large wavenumbers (Debnath, 2012; Ren et al., 2021). Furthermore, it is known that the BBM wave exhibits modulation instability during propagation (Halfiani et al., 2018; Zakharov, 2006). This modulation instability is one of the drivers of the emergence of extreme waves. As a result, BBM waves are both intriguing and promising in terms of generating extreme waves in the laboratory. The modulation frequency which is at modulation instability ( $0 < \tilde{\nu} < \sqrt{2}$ ) and phase singularity interval ( $0 < \tilde{\nu} < \sqrt{3/2}$ ) is an important input and it is necessary to find its value to avoid trial

and error when testing on the wave tank (Fadhiliani et al., 2020). The BBM wave can be studied using either the BBM equation (Qausar et al., 2019) or the envelope of the BBM wave group. The BBM wave group is known to evolve according to the Nonlinear Schrodinger equation (NLS) (Hu et al., 2015), and a phase singular phenomenon appears when the real value of the amplitude  $|A_{SFB}|$  disappears (Conforti et al., 2020; Wang and Wei, 2020). The presence of wave dislocation, as indicated by the merging or splitting of two waves, characterizes phase singularity. The amplitude of the wave changes significantly as a result of this phenomenon (Andonowati et al., 2007). The investigation of wave dynamics using the BBM group envelope, which is interpreted in the form of an Argand diagram, reveals the same thing: phase singularity appears in wave propagation (Fadhiliani et al., 2020). The Hamiltonian can also be used to explain this phase singularity. Hamiltonian can also explain the distribution of wave energy during its propagation because it contains potential energy (Sulem and Sulem, 1999). The wave group envelope equation, namely NLS, is used to derive this Hamiltonian formula. The aim of this research is to investigate the distribution of energy in each modulation frequency change. The modulation frequency is changed based on the value of the modulation instability interval, ie  $0 < \tilde{\nu} < \sqrt{2}$ . In this range there is also a phase singularity interval, ie  $0 < \tilde{\nu} < \sqrt{3/2}$ . It is done to improve knowledge of the energy characteristics of waves during their propagation and to determine the appropriate parameters for generating extreme waves in the laboratory. This study has been carried out in the Modeling and Simulation Laboratory, Department of Mathematics, Universitas Syiah Kuala, Indonesia in 2021.

## MATERIALS AND METHODS

### BBM Equation and wave group envelope

The Benjamin Bona Mahony (BBM) equation simulates one-way wave propagation with high wave numbers and low amplitudes. The energy formula takes using Eq. 1 (Benjamin et al., 1972).

$$\eta_t + \eta_x + \eta\eta_x - \eta_{xxt} = 0. \quad (1)$$

It is assumed that ansatz is periodic,  $\eta(x, t) = a(x, t)e^{i\theta} + c.c.$ , as a solution to the Eq. 1, where  $\theta = (kx - \omega t)$ ,  $a(x, t)$  express the amplitude of

the wave,  $k$  is the wavenumber,  $\omega$  is the frequency of the wave, and c.c. declares complex comrade.

The envelope or curve of the energy wave amplitude evolves according to the NLS equation, allowing these equations to be used to analyze the propagation of the energy wave envelope. Spatial NLS equations can be used as a mathematical model to predict the evolution of the envelope in space. The reduction in the BBM equation's spatial NLS equation is expressed using Eq. 2 (Slunyaev *et al.*, 2015).

$$A_{\xi} + i\hat{\beta}A_{\tau\tau} + \hat{\gamma}|A|^2A = 0, \quad (2)$$

where,  $A$  express the envelope amplitude, dispersive coefficient  $\hat{\beta} = -\left(2\omega p + \frac{\omega^2}{k}\right)/p^3$ , nonlinear coefficient  $\hat{\gamma} = \omega\left(\frac{2}{p-1} - \frac{k}{8k^2\omega + 2\omega - 2k}\right)/p$  with  $p = (1 + 2k\omega)/(1 + k^2)$  through the multiple scale method (Halfani *et al.*, 2017). Eq. (2) is obtained by applying the fast to slow variable transformation, where  $\theta = (kx - \omega t)$  as fast variable and slow variable;  $\xi = \varepsilon^2 x$ ,  $\tau = \varepsilon(t - x/p)$ ,  $a(x, t) = \varepsilon A(\xi, \tau)$ . Independent variables  $\xi$  and  $\tau$  express different meanings for different problems. In the case of dispersive waves,  $\xi$  describes spatial variables (space) and  $\tau$  represents the time variable.

Spatial NLS is appropriate for problems involving wave signals as the initial signal on the wave marker to describe space propagation. The NLS equation has numerous solutions that describe various phenomena. Soliton on Finite Background (SFB) is one of many NLS exact solutions that can describe extreme wave events in a hydrodynamics laboratory (Karjanto *et al.*, 2007).

#### SFB with displaced variables

The SFB solution is a non-linear interaction of an amplitude monochromatic signal  $r_0$ , i.e.  $r_0 e^{-i\gamma r_0^2 \tau}$ , which is disturbed by a modulating wave with a small  $\kappa$  wavenumber interval and results in an instability that increases exponentially with the rate  $\sigma(\kappa) = \kappa \sqrt{2r_0^2 \beta \gamma - \beta^2 \kappa^2}$ . A signal of this type is known as a Benjamin Feir (BF) signal (Benjamin and Feir, 1967; Karjanto *et al.*, 2011). The SFB wave signal was chosen because it can be generated with moderate

amplitude on the wavemaker. While the other solutions, Ma breather soliton (Mahato *et al.*, 2021), Akhmediev breather soliton (González-Gaxiola and Biswas, 2018) and the rational solution, all solutions describe the finite background wave type, they are not suitable as wavemaker inputs in practice. The Ma soliton wave signal cannot be used because it requires a maximum amplitude as input to the wavemaker, and the rational solution is difficult to use because the rational wave signal has an infinite modulation period, or it is not periodic with respect to space or time (Karjanto, 2006). The SFB solution to the spatial NLS, using Eq. 3.

$$A(\xi, \tau) = A_{SFB}(\xi, \tau) r_0 e^{-i\gamma r_0^2 \xi}, \quad (3)$$

Where,

$$A_{SFB}(\xi, \tau) = \frac{(\hat{v}^2 - 1) \cosh(\sigma(v)\xi) + \sqrt{1 - \frac{\hat{v}^2}{2}} \cos(v\tau) - i\hat{v}\sqrt{2 - \hat{v}^2} \sinh(\sigma(v)\xi)}{\cosh(\sigma(v)\xi) - \sqrt{1 - \frac{\hat{v}^2}{2}} \cos(v\tau)}$$

and  $v$  states the modulation frequency,  $\sigma(v) = v\sqrt{2r_0^2 \hat{\beta} \hat{\gamma} - \hat{\beta}^2 v^2}$  and  $\hat{v} = v/(r_0 \sqrt{\hat{\gamma}/\hat{\beta}})$ .

Displaced phase-amplitude variables are the results of the transformation of SFB variables that were originally in real form into complex forms (van Groesen *et al.*, 2006). Its purpose is to investigate changes in amplitude in complex planes with phase that is solely dependent on position. This NLS solution is derived based on variational formula depends on phase  $\phi$ . With  $(\xi, \tau) = (0, 0)$  as the maximum position and time (Fig. 1).

As a result of the change in phase with respect to position, the wavelength of the carrier waves from the wave group changes, and this becomes a driving force towards extreme waves. SFB form with displaced phase-amplitude variables is given using Eq. 4.

$$A(\xi, \tau) = A_0(\xi) F(\xi, \tau), \quad (4)$$

where,  $F(\xi, \tau) = G(\xi, \tau) e^{i\phi(\xi, \tau)} - 1$ ,  $\phi(\xi, \tau)$  as a

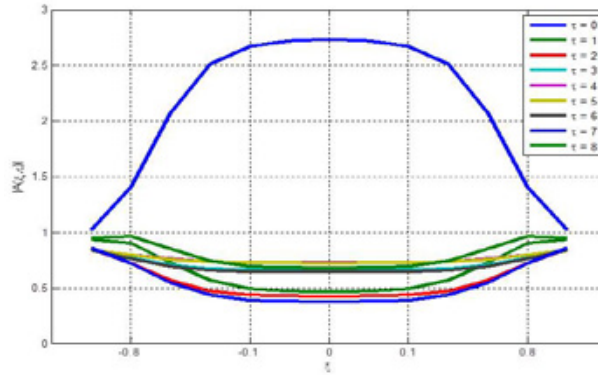


Fig. 1: Graph of envelope  $|A_{SFB}|$  for  $(\tilde{r}_0, \beta, \gamma, \tilde{v}) = (1, 1, 1, \sqrt{1/2})$  (Fadhiliani et al., 2019)

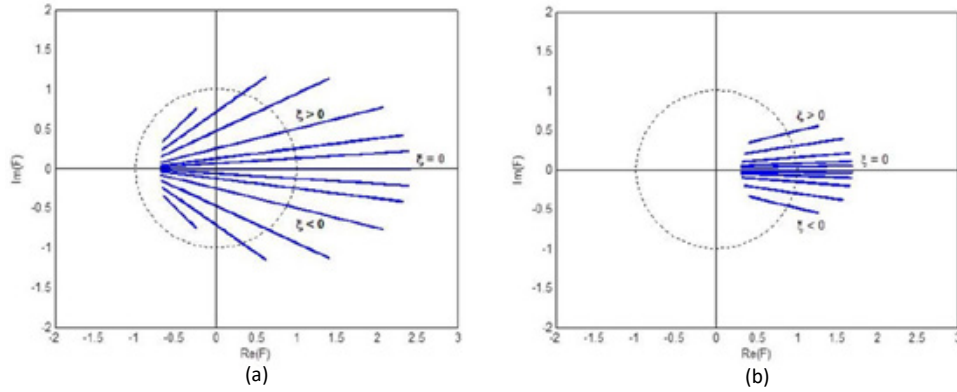


Fig. 2: Argand diagram SFB for  $(\tilde{r}_0; \tilde{k}) = (0.2; 0.9186)$ ; (a)  $\tilde{v} = 1$  and (b)  $\tilde{v} = 1.3$  (Fadhiliani et al., 2020)

replacement phase, and  $G(\xi, \tau)$  as a replacement amplitude.

The phase singularity phenomenon can be proven by transforming the SFB variable into a complex form and presenting the results in an Argand diagram. When the modulation frequency is in the interval  $[0, \sqrt{3/2}]$ , the phenomenon of phase singularity occurs. This interval is represented by a straight line that passes through the origin twice (Fig. 2a). Meanwhile, for waves with modulation frequencies in the modulation instability  $(0, \sqrt{2})$  interval but not in the phase singularity interval, the straight line on the Argand diagram passes through the origin only once (Fig. 2b). A significant increase in amplitude is caused by modulation instability, which is a mechanism for extreme wave and phase singularity phenomena.

#### Hamiltonian formula

In Lagrangian form, the NLS equation is a dynamic system. The integral of the equation contains a large number of quantities. One of the quantities in question is Hamiltonian, but there is also wave energy, mass, or wave power in optics, as well as a conserved quantity known as momentum (linear). The equations for wave-ship interactions are based on the Lagrangian variational principle, which results in the combined system being formulated as a Hamiltonian system (van Groesen et al., 2017). Because it contains potential energy and proves singularity for finite time in the NLS equation, the Hamiltonian plays a role in regulating phase amplitude dynamics.

The Hamiltonian is derived from the Lagrangian according to the NLS equation which satisfies the evolution of the BBM wave envelope. The exact solution from NLS was chosen, namely SFB because it is able to

explain the dynamics of wave propagation that have modulation instability and is suitable for input signals to wavemakers in the laboratory. The SFB transformation uses displaced variables so that they can be interpreted geometrically, and these variables are also used in Hamiltonian. The Hamiltonian for NLS, using Eq. 5.

$$H(A) = \int_{-\infty}^{\infty} \left( -\frac{1}{2} \beta |\partial_{\tau} A|^2 + \frac{1}{4} \gamma |A|^4 \right) d\tau, \quad (5)$$

so that the Hamiltonian transformation  $H(G, \phi)$  is obtained which contains the displaced phase-amplitude variable using Eq. 6.

$$H(G, \phi) = \int_{-T/2}^{T/2} \left( -\frac{1}{2} \beta r_0^2 \left[ (\partial_{\tau} G)^2 + G^2 (\partial_{\tau} \phi)^2 \right] + \frac{1}{4} \gamma r_0^4 (G(G - 2 \cos \phi) + 1)^2 \right) d\tau. \quad (6)$$

The Hamiltonian equation  $H(G, \phi)$  containing the displaced phase-amplitude variable begins with

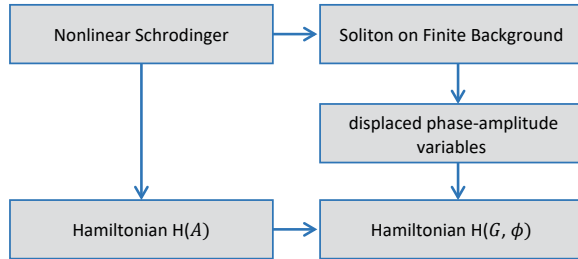


Fig. 3. Schematic diagram of methods

the spatial NLS equation and is not time dependent. Because the Hamiltonian for this system is not time dependent, Hamiltonian represents total energy as the sum of kinetic and potential energies and that it is independent of time (Akhmediev *et al.*, 1997; Eisberg and Resnick, 1985). The total energy is conserved and is the sum of the kinetic and potential energies. Hamiltonian provides a phase-space integrated solution that is good for equations of motion and can also be interpreted geometrically (Karjanto, 2006).

## RESULTS AND DISCUSSION

The Hamiltonian function is used to investigate the distribution of BBM wave energy during its propagation. Because it contains potential energy, the Hamiltonian plays a role in regulating phase amplitude dynamics and also in proving singularity at finite time in the NLS-BBM equation. Previously, the Hamiltonian form  $H(G, \phi)$  containing displaced phase-amplitude variables was obtained. Because the Hamiltonian for this system does not depend on time, it will be equal to the mechanical energy or total energy, which is the sum of the kinetic and potential energies. The variables and parameters considered are non-dimensional, so the presentation of figures does not use units.

The results for normalized conditions are presented visually in Fig. 4. The modulation frequency is  $\tilde{\nu} = \sqrt{1/2}$  in the interval of modulation instability, and other parameters are 1. The red curve in Fig. 4 represents potential energy, the green curve represents kinetic energy, and the blue curve represents the Hamiltonian. It was found that the curve has symmetry with respect to position as shown

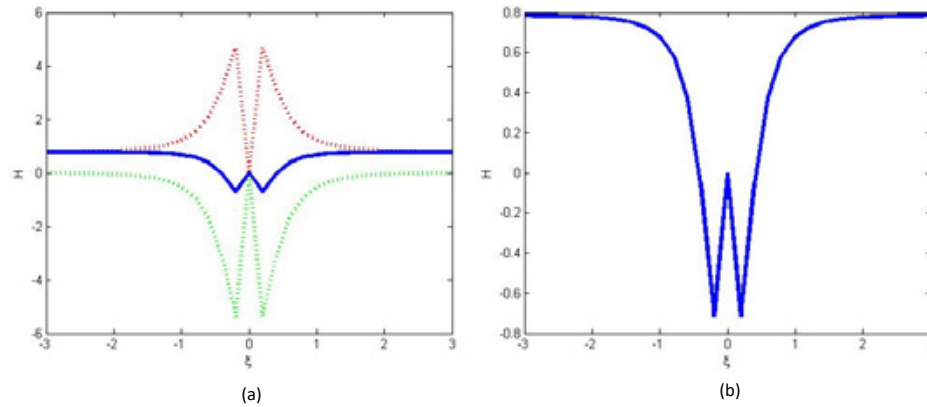


Fig. 4: (a) Energy distribution and (b) Hamiltonian's curve for normalized  $(\tilde{r}_0, \beta, \gamma, \tilde{\nu}) = (1, 1, 1, \sqrt{1/2})$

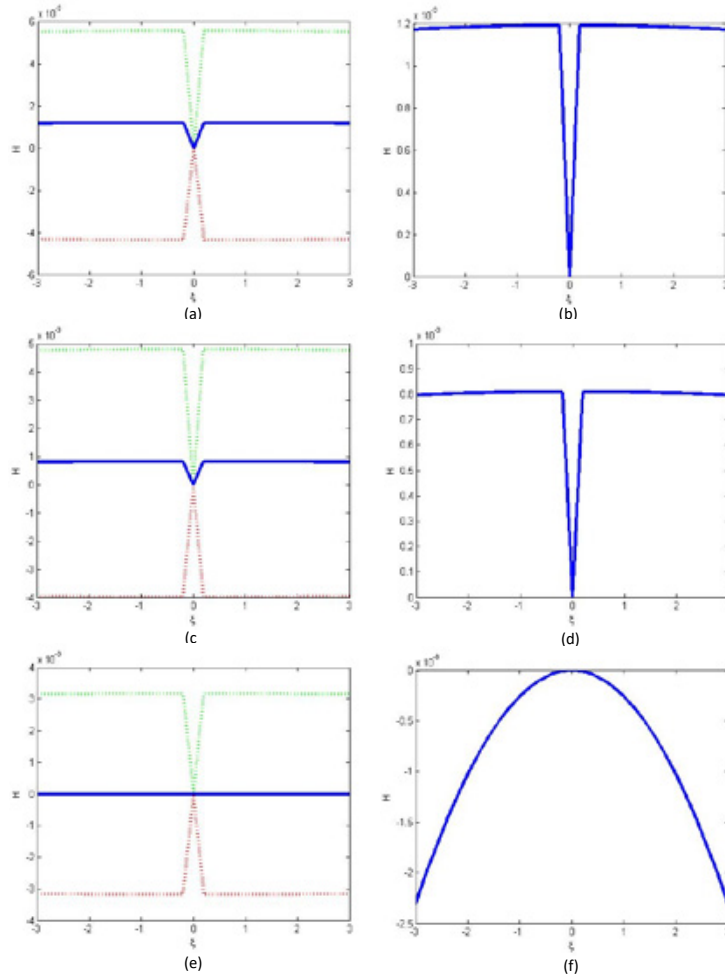


Fig. 5 : Energy distribution (left column) and Hamiltonian's curve (right column) for  $(\tilde{\tau}_0; \tilde{k}) = (0.2; 0.9186)$ ; (a) - (b)  $\tilde{\nu} = 0.5$ , (c) - (d)  $\tilde{\nu} = 0.7$ , and (e) - (f)  $\tilde{\nu} = 1$

in the graph of the envelope (Fig. 1). The resulting energy  $H$  has the same value on the negative and positive sides of the variable of position in space  $\xi$ . The shape of the symmetry is due to the symmetry envelope generated by the normalized frequency. This is an early sign that the distribution of wave energy in its propagation affects the amplitude and phase angle of the wave.

Fig. 5 contains the energy distribution and Hamiltonian curve for the modulating frequency at interval of the phase singularity phenomenon, it is  $0 < \tilde{\nu} < \sqrt{3/2}$ . In this frequency group, the curve also has a symmetrical shape with respect to position and has an optimum value when  $\xi = 0$  according to the initial assumption. For modulation frequency  $\tilde{\nu} = 0.5$

and  $\tilde{\nu} = 0.7$ , kinetic energy has a larger portion than potential energy, both are almost the same when the modulation frequency  $\tilde{\nu} = 1$ .

The energy distribution and Hamiltonian curves presented in Fig. 6 for the modulation frequency are outside the phase singularity interval but in the modulation instability interval, ie  $0 < \tilde{\nu} < \sqrt{2}$ . The curve has a symmetrical shape with respect to position and peak occurs when  $\nu$ . Potential energy has a larger portion than kinetic energy when the modulation frequency  $\tilde{\nu} = 1.3$  and  $\tilde{\nu} = 1.4$ , but is not greater than the potential energy in the wave with less frequency.

Potential energy is energy related to position due to the influence of gravity. When viewed physically in the study of surface water waves, potential energy is



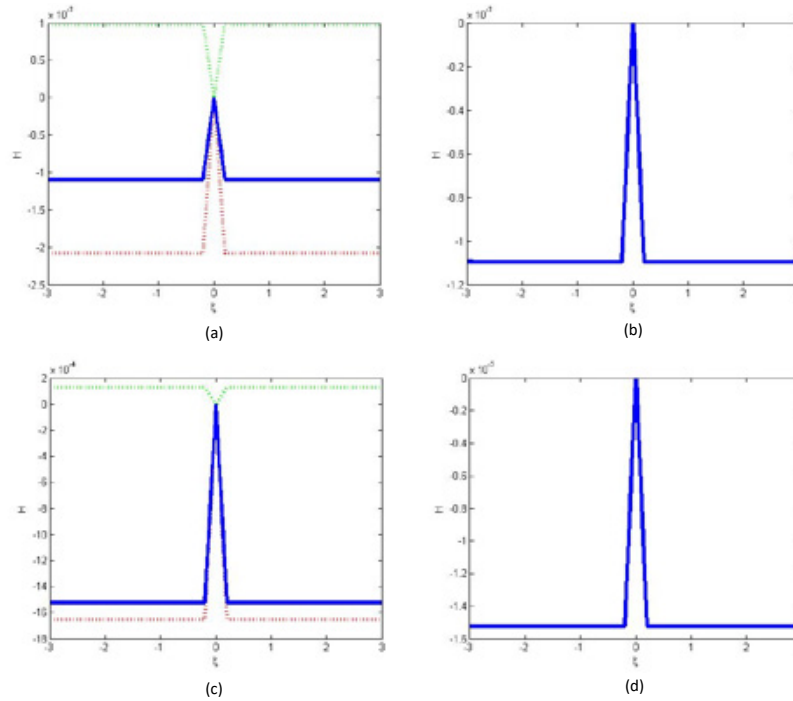


Fig. 6 : Energy distribution (left column) and Hamiltonian's curve (right column) for  $(\tilde{r}_0; \tilde{k}) = (0.2; 0.9186)$ ;  
(a) - (b)  $\tilde{\nu} = 1.3$  and (c) - (d)  $1.4$

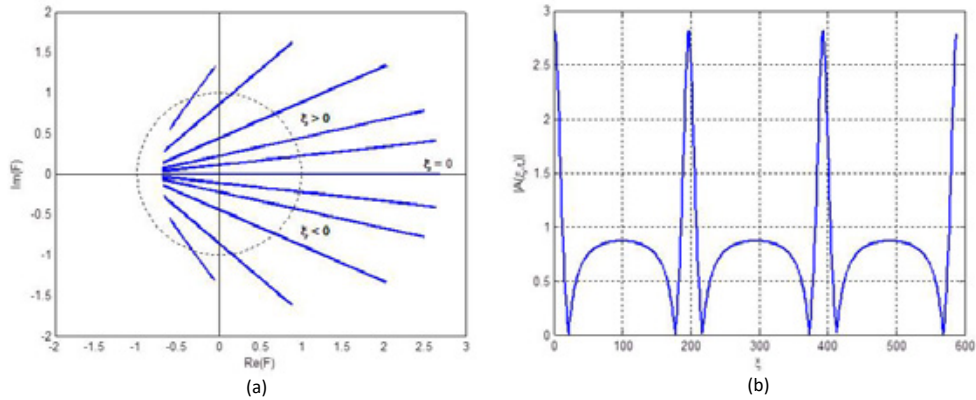


Fig. 7 : (a) Argand diagram and (b) envelope, for  $(\tilde{r}_0; \tilde{k}; \tilde{\nu}) = (0.2; 0.9186; 0.5)$

related to the wave amplitude. On the other hand, there is kinetic energy related to the wave motion to reach a certain speed and physically related to the steepness of the wave. Based on Figs. 5 and 6, the largest potential energy and kinetic energy are in the wave with the modulation frequency  $\tilde{\nu} = 0.5$  which is the smallest modulation frequency in this study.

The wave amplitude for the modulating frequency

$\tilde{\nu} = 0.5$  is shown by the envelope curve (Fig. 7). The amplitude for that frequency has a greater value than the wave amplitude at other modulating frequencies. Meanwhile, the phase angle with the same modulation frequency is shown through the Argand diagram (Fig. 7a) and is found to have a greater value than the phase angle at other modulating frequencies (the envelope curve and Argand diagrams for other frequencies can

be seen in Figs. 1 and 2). Thus, it is obtained that the amplitude will be smaller for the greater the modulation frequency. Similarly, the phase angle will be smaller for the greater the modulation frequency and the greater the phase angle, the steeper the wave will be. The generated wave (extreme) is a high wave and not broken. The use of a smaller modulation frequency seems to have to be decided with great consideration. High waves are needed to test the durability of the model ship in the test pool, but breaking waves should be avoided because they can damage laboratory facilities. More detailed observations are needed to see how the energy distribution relates to other parameters. They are wave number and initial amplitude in order to determine the combination of parameters to be used in order to obtain a wave with a maximum height and can maintain its shape.

## CONCLUSION

The distribution of energy in wave propagation is studied to complete information about the wave characteristics that will be used as the initial signal input to the wavemaker. The characteristics of the waves that have been obtained are used to decide whether certain mathematical models in this case BBM are suitable to be applied in the hydrodynamics laboratory in order to obtain extreme waves or at least approach them with all their limitations. This study simply computes some signal input modulation frequency in wave generation. This is for efficient use of the laboratory, avoiding the practice of trial and error. Based on the information that has been obtained previously, that the BBM wave experiences modulation instability for the modulation frequency at intervals of  $0 < \tilde{\nu} < \sqrt{2}$  which causes amplitude amplification. Then the phase singularity phenomenon also appears in wave propagation which gives a significant increase in amplitude for the modulation frequency interval  $0 < \tilde{\nu} < \sqrt{3/2}$ . It is interesting to see the energy distribution of the BBM wave in the modulation frequency interval. Singularities and energy distributions can be described by the Hamiltonian. The Hamiltonian used contains displaced-phase amplitude in a complex plane with a phase that only depends on position, so that the Hamiltonian will be equal to the total energy which is the sum of the potential energy and kinetic energy. The modulation frequency considered as an extreme wave generator is  $\tilde{\nu} = 0.5$ , because the resulting amplitude is quite high and the energy in the envelope is also quite large. Hamiltonian curve is symmetrical with respect to

position. The modulation frequency affects the amount of energy that participates in wave propagation. The smaller the modulation frequency, the greater the potential energy and kinetic energy. Potential energy is related to the amplitude and kinetic energy is related to the steepness of the wave so that the amplitude and steepness of the wave will be maximum for small modulating frequencies. In practice, even if a wave with a maximum amplitude is desired, the selection of a smaller modulation frequency must be considered because it will have an impact on the steepness of the resulting wave. Other parameters, namely wave number and initial amplitude, are limitations in this paper, so it can be a concern to get closer to the goal of extreme waves in the laboratory.

## AUTHOR CONTRIBUTIONS

D. Fadhiliani performed the literature review, running the model, analyzed and interpreted the data, prepared the manuscript text, and manuscript edition. M. Ikhwan performed the literature review, prepared the manuscript text, and manuscript edition. M. Ramli performed the literature review, analyzed and interpreted the data, prepared the manuscript text, and manuscript edition. S. Rizal performed the literature review, prepared the manuscript text, and manuscript edition. M. Shafwan performed the literature review, prepared the manuscript text, and manuscript edition.

## ACKNOWLEDGEMENT

This study is funded by Universitas Syiah Kuala, Banda Aceh, Ministry of Education, Culture, Research and Technology, Indonesia, under the scheme of "PRUUPD", Universitas Syiah Kuala with contract number [357/UN11.2.1/PT.01.03/PNBP/2021].

## CONFLICT OF INTEREST

The authors declare no potential conflict of interest regarding the publication of this work. In addition, the ethical issues including plagiarism, informed consent, misconduct, data fabrication and, or falsification, double publication and, or submission, and redundancy have been completely witnessed by the authors.

## OPEN ACCESS

This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction



in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons license, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons license and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this license, visit: <http://creativecommons.org/licenses/by/4.0/>.

## ABBREVIATIONS

$A$	Equation of envelope amplitude
$ A $	Modulus of complex number $A$
$A_0$	Plane wave solution of the NLS equation
$a$	Equation of amplitude
$BBM$	Benjamin Bona Mahony
$BF$	Benjamin Feir
$\beta$	Dispersion coefficient of the NLS equation
$\hat{\beta}$	Dispersion coefficient of the NLS Spatial equation
$c.c.$	Conjugate complex
$Eq.$	Equation
$e$	Euler's Number, $e = 2,71828...$
$\varepsilon$	Small positive real number parameter
$\eta$	Wave elevation
$F$	Complex amplitude of waves on finite background
$Fig.$	Figure
$G$	Real-valued displaced amplitude
$\gamma$	Nonlinear coefficient of the NLS equation
$\hat{\gamma}$	Nonlinear coefficient of the NLS spatial equation
$H$	Hamiltonian
$i$	Imaginary number, $i = \sqrt{-1}$
$KdV$	Koerteweg de Vries
$K$	Modulation wavenumber
$k$	Dimensionless wave number
$\tilde{k}$	Dimensional wavenumber
$NLS$	Nonlinear Schrödinger
$\nu$	Modulation frequency
$\hat{\nu}$	Normalized $\nu$
$\tilde{\nu}$	Dimensional modulation frequency
$\omega$	Dimensionless wave frequency;

$\tilde{\omega}$	Dimensional wave frequency
$p$	Phase of carrier wave
$\phi$	Displaced phase
$r_0$	The initial amplitude of the dimensionless wave
$\tilde{r}_0$	The initial amplitude of the dimension wave
$\sigma$	Instability level
$SFB$	Soliton on Finite Background
$T$	Period
$t$	Time variable
$\tau$	Time slow variable
$\theta$	Phase of monochromatic wave
$x$	Spatial variables
$\xi$	Spatial slow variable

## REFERENCES

- Akhmediev, N.N.; Ankiewicz, A., (1997). Solitons-nonlinear pulses and beams. Chapman and Hall.
- Andonowati; Karjanto, N.; van Groesen, E., (2007). Extreme wave phenomena in down-stream running modulated waves. Appl. Math. Modell., 31(7): 1425–1443 (19 pages).
- Benjamin, T.B.; Feir, J.E., (1967). The disintegration of wave trains on deep water Part 1. Theory. J. Fluid Mech., 27: 417-430 (14 pages).
- Benjamin, T.B.; Bona, J.L.; Mahony, J.J., (1972). Model equations for long waves in nonlinear dispersive systems. Philos. Trans. Roy. Soc. London, Ser. A, 272: 47-57 (11 pages).
- Conforti, M.; Mussot, A.; Kudlinski, A.; Trillo, S.; Akhmediev, N., (2020). Doubly periodic solutions of the focusing nonlinear Schrödinger equation: Recurrence, period doubling, and amplification outside the conventional modulation-instability band. Phys. Rev. A, 101(023843): 1-11 (11 pages).
- Dean, R.G., (1990). Freak waves: A possible explanation. In: Torum A, Gudmestad OT (eds) Water Wave Kinetics. Kluwer.
- Debnath, L., (2012). Nonlinear partial differential equations for scientists and engineers, 4th edition. Birkhäuser.
- Didenkulova, E., (2020). Catalogue of rogue waves occurred in the World Ocean from 2011 to 2018 reported by mass media sources. Ocean Coastal Manage., 188(105076): 1-20 (20 pages).
- Fadhiliani, D.; Ramli, M.; Rizal, S.; Syafwan, M., (2019). Wave properties using displaced phase amplitude. IOP Conf. Ser.: Earth Environ. Sci., 364: 1-6 (6 pages).
- Fadhiliani, D.; Halfiani, V.; Ikhwan, M.; Qausar, H.; Munzir, S.; Rizal, S.; Syafwan, M.; Ramli, M., (2020). The dynamics of surface wave propagation based on the benjamin bona mahony equation. Heliyon, 6(e04004): 1-10 (10 pages).
- González-Gaxiola, O.; Biswas, A., (2018). Akhmediev breathers, peregrine solitons and kuznetsov-ma solitons in optical fibers and pcf by laplace-adomian decomposition method. Optik, 172: 930-939 (10 pages).
- Halfiani, V.; Salmawaty; Ramli, M., (2017). An envelope equation of benjamin bona mahony wave group. Far. East J. Math. Sci., 102(6): 1077–1090 (14 pages).
- Halfiani, H.; Fadhiliani, D.; Mardi, H.A.; Ramli, M., (2018). Nonlinear evolution of benjamin-bona-mahony wave packet due to an instability of a pair of modulations. Int. J. Differ. Equations. 2018(1716571): 1-10 (10 pages).
- Horn, D.A.; Imberger, J.; Ivey, G.N., (1999). Internal solitary waves in lakes – a closure problem for hydrostatic models. Proc. Aha Huliko

- Hawaiian Winter Workshop—Internal Gravity Waves II, 95-101 (7 pages).
- Hu, Z.; Tang, W.; Xu, H.; Zhang, X., (2014). Numerical study of rogue waves as non-linear schrodinger breather solutions under finite water depth. *Wave Motion*, 52: 81-90 (10 pages).
- Ikhwan, M.; Wafdan, R.; Haditir, Y.; Ramli, M.; Muchlisin, Z.A.; Rizal, S., (2021). Hydrodynamics and tidal turbine generator stability analysis in several wave variations. *Int. J. Differ. Equations*, 2021(6682407): 1-8 (8 pages).
- Karjanto, N., (2006). Mathematical aspects of extreme water waves. University of Twente.
- Karjanto, N.; van Groesen, E., (2007). Note on wavefront dislocation in surface water waves. *Phys. Lett. A*, 371: 173-179 (7 pages).
- Karjanto, N.; van Groesen, E., (2010). Qualitative comparisons of experimental results on deterministic freak wave generation based on modulational instability. *J. Hydro-Environ. Res.*, 3: 186-192 (7 pages).
- Karjanto, N.; van Groesen, E.; Peterson, P., (2011). Investigation of the maximum amplitude increase from the benjamin feir instability. *Physics. Flu. dyn.*, 1-9 (9 pages).
- Korteweg, D.J.; de Vries, G., (1895). On the change of form of long waves advancing in a rectangular canal, and on a new type of long stationary waves. *Philos. Mag. A*, 39(5): 422-443 (22 pages).
- Mahato, D.K.; Govindarajan, A.; Lakshmanan, M.; Sarma, A.K., (2021). Dispersion managed generation of peregrine solitons and kuznetsov-ma breather in an optical fiber. *Phys. Lett. A*, 392: 127134 (16 pages).
- Myint-U.T.; Debnath, L., (2007). Linear partial differential equations for scientists and engineers, 4th edition, Birkhäuser.
- Qausar, H.; Zahara Y.; Ramli, M.; Munzir, S.; Halfiani, V., (2019). Nonlinear evolution of bichromatic waves based on the fifth order solution of benjamin bona mahony equation. *J. Phys. Conf. Ser.*, 1566. 012091: 1-10 (10 pages).
- Ren, J.; Ilhan, O.A.; Bulut, H.; Manafian, J., (2021). Multiple rogue wave, dark, bright, and solitary wave solutions to the KP–BBM equation. *J. Geom. Phys.*, 164(104159): 1-18 (18 pages).
- Eisberg, R.; Resnick, R., (1985). Quantum Physics of Atoms, Molecules, Solids, Nuclei and Particles (2nd ed.). John Wiley and Sons.
- Samaras, A.G.; Karambas, T.V., (2021). Numerical simulation of ship-borne waves using a 2DH post-Boussinesq model. *Appl. Math. Modell.*, 89: 1547-1556 (10 pages).
- Shiralashetti, S.C.; Hanaji, S.I., (2021). Hermite wavelet based numerical method for the solution of two parameters singularly perturbed non-linear Benjamina-Bona-Mohany equation. *Sci. African.*, 12(e00770): 1-8 (8 pages).
- Slunyaev, A.; Sergeeva, A.; Pelinovsky, E., (2015). Wave amplification in the framework of forced non linear schrodinger equation: the rogue wave context. *Physica .D*, 303: 18-27 (10 pages).
- Sulem, C.; Sulem, P.L., (1999). The nonlinear schrödinger equation: self focusing and wave collapse. Springer-Verlag.
- van Groesen, E.; Andonowati; Karjanto, N., (2006). Displaced phase-amplitude variables for waves on finite background. *Phys. Lett. A*, 354(4): 312–219 (8 pages).
- van Groesen, E.; Turnip, P.; Kurnia, R., (2017). High waves in Draupner seas—Part 1: numerical simulations and characterization of the seas. *J Ocean Eng. Mar. Energy*, 3(3): 233–245 (13 pages).
- Wang, J.; Ma, Q.W.; Yan, S.; Qin, H., (2018). Numerical study on the quantitative error of the korteweg–de vries equation for modelling random waves on large scale in shallow water. *Eur. J. Mech. B. Fluids*, 71: 92-102 (11 pages).
- Wang, X.; Wei, J., (2020). Antidark solitons and soliton molecules in a (3 + 1)-dimensional nonlinear evolution equation. *Nonlinear Dyn.*, 102(1): 363–377 (15 pages).
- Waseda, T.; Tamura, H.; Kinoshita, T., (2021). Freakish sea index and sea states during ship accidents. *J. Mar. Sci. Tech.*, 17: 305-314 (10 pages).
- Zhao, X.; Hu, C.; Sun, Z., (2010). Numerical simulation of extreme wave generation using vof method. *J. Hydrodyn.*, 22: 466–477 (12 pages).
- Zakharov, V.E.; Dyachenko, A.I.; Prokofiev, A.O., (2006). Freak waves as nonlinear stage of Stokes wave modulation instability. *Eur. J. Mech. B. Fluids*, 25(5): 677-692 (16 pages).

#### AUTHOR (S) BIOSKETCHES

**Fadhiliani, D.**, Ph.D. Candidate, Assistant Professor, Graduate School of Mathematics and Applied Sciences, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia. Email: [dwifadhiliani@gmail.com](mailto:dwifadhiliani@gmail.com)  
ORCID: 0000-0001-5775-4346

**Ikhwan, M.**, Ph.D. Candidate, Assistant Professor, Graduate School of Mathematics and Applied Sciences, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia. Email: [m.ikhwan@mhs.unsyiah.ac.id](mailto:m.ikhwan@mhs.unsyiah.ac.id)  
ORCID: 0000-0002-8162-1479

**Ramli, M.**, Ph.D., Professor, Department of Mathematics, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia.  
Email: [marwan.math@unsyiah.ac.id](mailto:marwan.math@unsyiah.ac.id)  
ORCID: 0000-0003-1225-9063

**Rizal, S.**, Ph.D., Professor, Department of Marine Sciences, Universitas Syiah Kuala, Banda Aceh 23111, Indonesia.  
Email: [srizal@unsyiah.ac.id](mailto:srizal@unsyiah.ac.id)  
ORCID: 0000-0003-3637-2351

**Syafwan, M.**, Ph.D., Associate Professor, Department of Mathematics, Universitas Andalas, Padang 25163, Indonesia.  
Email: [mahdhivan@sci.unand.ac.id](mailto:mahdhivan@sci.unand.ac.id)  
ORCID: 0000-0003-2907-3644

#### HOW TO CITE THIS ARTICLE

Fadhiliani, D.; Ikhwan, M.; Ramli, M.; Rizal, S.; Syafwan, M., (2022). Distribution of energy in propagation for ocean extreme wave generation in hydrodynamics laboratory. *Global J. Environ. Sci. Manage.*, 8(1): 17-26.

DOI: 10.22034/gjesm.2022.01.02

url: [https://www.gjesm.net/article\\_244655.html](https://www.gjesm.net/article_244655.html)

